
Modelling the Doppler Effect and Mach Cone Formation

A Technical Report on an Interactive Simulation of Subsonic, Sonic,
and Supersonic Wave Behaviour

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Abstract

The Doppler effect is the apparent change in a wave's observed frequency caused by relative motion between its source and an observer. This report focuses on sound emitted by a moving source and detected by a stationary observer in a stationary medium, and investigates how the relationship between a supersonic aircraft's velocity and the local speed of sound dictates the formation of a Mach cone. An interactive digital simulation was created as a learning tool for Year 11 Physics students. Users can vary source speed, wave speed, and emitted frequency continuously and immediately see changes in compression, rarefaction, observed frequency and Mach cone geometry. The model allows experimentation across subsonic, sonic and supersonic conditions, whilst its quantitative readout calculates the source velocity, Mach number and observed frequencies. Although the simulation depicts 2D kinematics effectively, it is restricted to a two-dimensional planar surface and assumes a homogeneous atmosphere. Future versions could add an opacity gradient to approximate intensity attenuation under the Inverse Square Law and a vertical temperature gradient to simulate atmospheric refraction. Nevertheless, the model remains effective because its numerical feedback and interactivity help students connect abstract Doppler-effect equations to observable wavefront behaviour.

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1 Introduction

This report focuses on the Doppler effect and investigates how the relationship between a supersonic aircraft's velocity and the local speed of sound dictates the formation of a Mach cone.

1.1 Physics Principles and Modern Application

The Doppler effect, which occurs in both mechanical and electromagnetic waves, is the apparent change in a wave's observed frequency caused by relative motion between its source and an observer (Halliday et al., 2014). This report focuses on sound emitted by a moving source and detected by a stationary observer in a stationary medium. When the source speed v_s is less than the local speed of sound v_w , wavefronts compress ahead of the source, resulting in a shorter wavelength and higher observed frequency, perceived as a higher pitch. Simultaneously, the waves behind it rarefy to produce a longer wavelength, lower frequency and lower pitch (Ling et al., 2016). For this situation,

$$f' = f \frac{v_w}{v_w \pm v_s}, \quad (1)$$

where f is the emitted frequency and f' is the observed frequency. The minus sign applies when the source approaches the observer, and the plus sign as it recedes (Dommel et al., 2018; Ling et al., 2016).

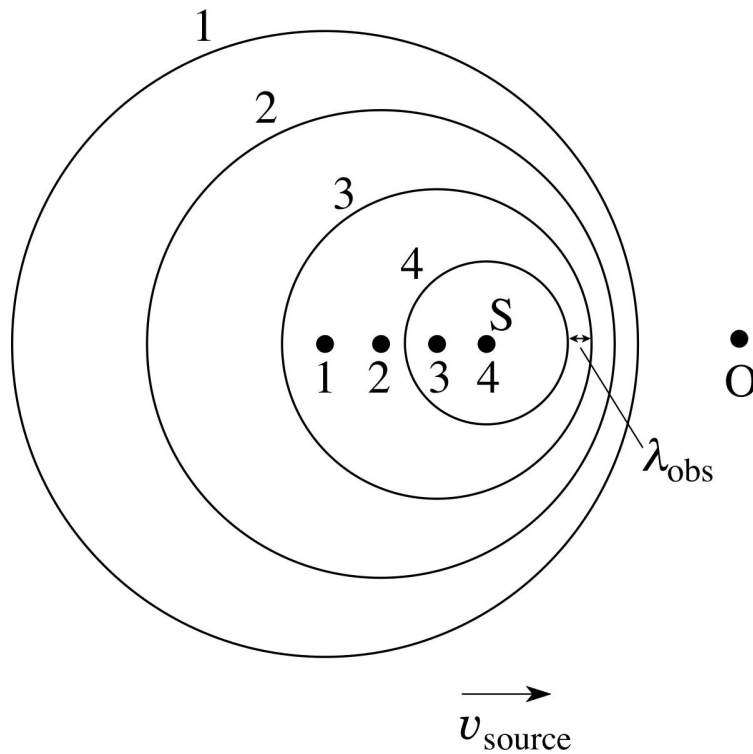


Figure 1. Wavefront compression and rarefaction around a moving subsonic source.

Note. Adapted from Open University (1996).

However, when the source accelerates past the local speed of sound ($v_s > v_w$), a different wave behaviour appears. At exactly Mach 1 ($v_s = v_w$), the source travels at the same speed as the forward wavefronts, which results in the formation of a densely compressed region where wavefronts coalesce (Anderson, 2017). Once in the supersonic regime, the source travels faster than its own propagating waves.

According to Huygens' Principle (Ling et al., 2016), every point on the wavefront acts as a secondary source, so as the source is travelling faster than these expanding secondary wavelets, the waves constructively interfere and form a continuous shock wave boundary known as the Mach cone. Its geometry is determined by the Mach angle (μ), calculated using the relationship:

$$\sin(\mu) = \frac{v_w}{v_s}. \quad (2)$$

As the source velocity increases, the ratio v_w/v_s decreases, thereby reducing the Mach angle μ . The narrower geometry causes the Mach cone to intersect the ground further behind the aircraft, so ground observers hear the sonic boom after the aircraft has already passed overhead. The relationship and resulting wavefront geometry can be seen in Fig. 2.

The wave behaviour has major implications for aerospace engineering and supersonic aviation. When aircraft such as the Concorde or fighter jets travel at supersonic speeds, they generate a continuous Mach cone (Anderson, 2017) that trails behind the aircraft and intersects with the ground. For observers on the ground, the sudden pressure change produced as the shock wave passes overhead is perceived as a sonic boom, as seen in Fig. 3. Consequently, engineers must model sonic-boom pressure signatures when designing airframes and flight paths to reduce overpressure and noise, thereby limiting damage to infrastructure below (Cliff et al., 2015).

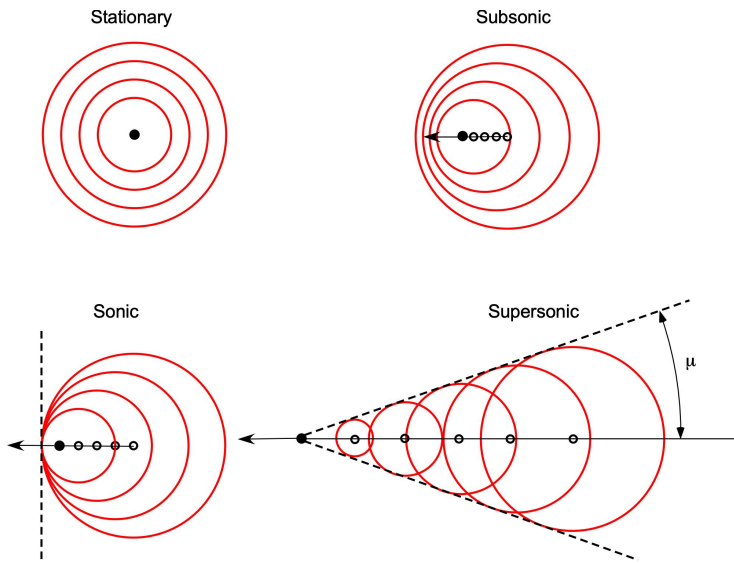


Figure 2. A moving point source illustrates compressibility effects in flow.

Note. Adapted from Leishman (2022).

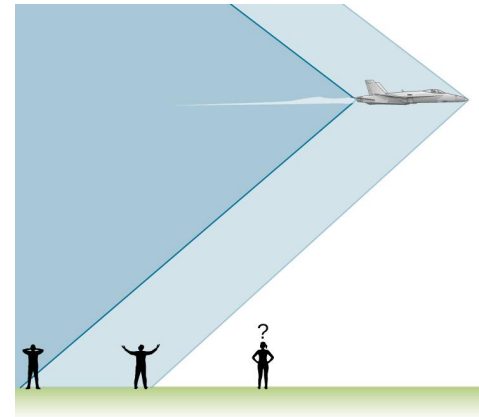


Figure 3. Two sonic booms experienced by observers, created by the nose and tail of an aircraft as the shock wave sweeps along the ground, are observed on the ground after the plane has passed by.

Note. Adapted from Ling et al. (2016).

2 Model Demonstration and Annotations

The interactive model is available at <https://dopplersim.ruben-roy.com/>.

2.1 User Interface and Layout

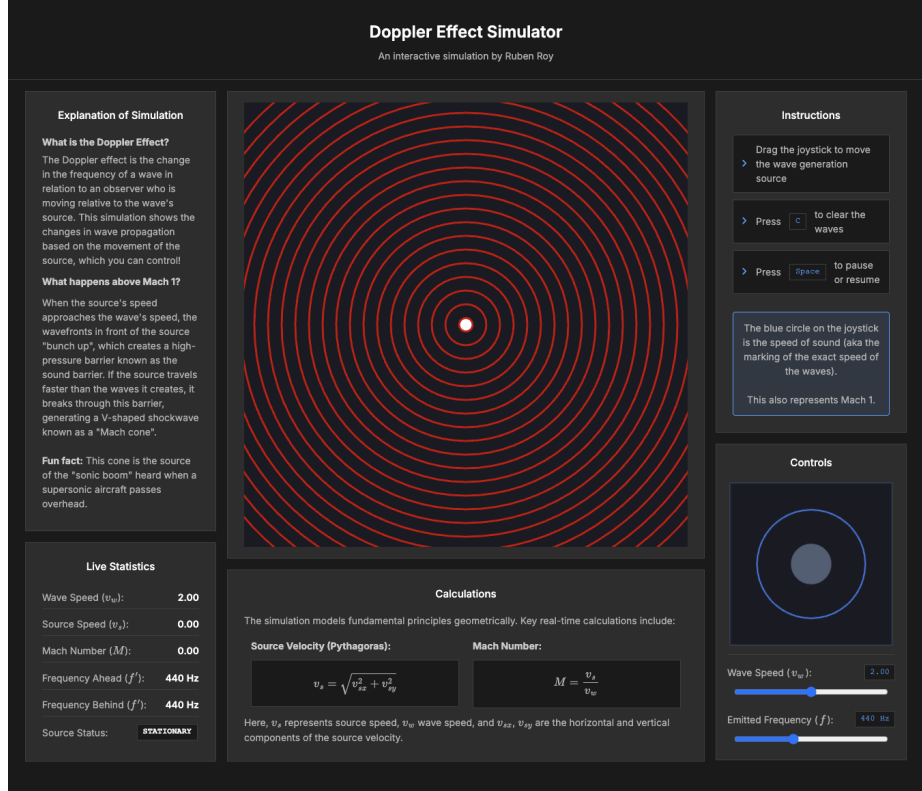


Figure 4. User interface and layout.

Note. Screenshot by the author.

The website is split into six components. The main canvas displays the wavefronts generated by the moving source in real time. On the left, the Explanation of Simulation panel explains the relevant physics, whilst the Live Statistics panel continuously displays the wave speed (v_w), source speed (v_s), Mach number (M), frequencies ahead and behind the source (f'), and the resulting source state.

To the right, the Instructions panel explains the keyboard and joystick inputs, and the Controls panel allows the user to change the source velocity, wave propagation speed and emitted frequency (f). The blue perimeter of the joystick marks the velocity required for Mach 1, which lets users directly compare between v_s and v_w . Adjusting v_w also allows students to simulate different propagation media in which sound travels differently (e.g., water). Increasing it expands the blue Mach 1 perimeter, showing that a higher source velocity is required to become supersonic in a faster medium.

Under the canvas, the Calculations panel shows how the source velocity is found from its horizontal and vertical components using Pythagoras, and how the Mach number (M) is calculated. Such an arrangement allows the displayed wave behaviour to be immediately compared with its mathematical cause by keeping both the visual model and quantitative evidence visible at the same time.

2.2 Subsonic Doppler Shift ($M < 1$)

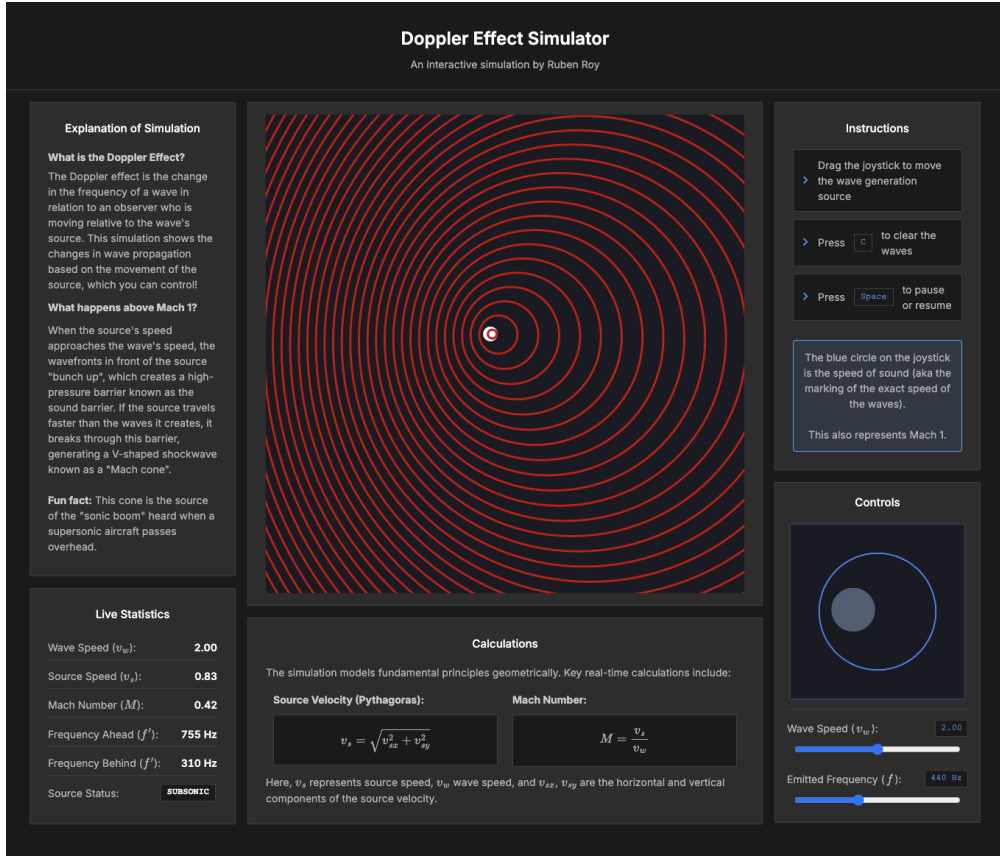


Figure 5. Subsonic Doppler shift ($M < 1$).

Note. Screenshot by the author.

This screenshot shows the simulation rendering the wave pattern of a classical Doppler shift. As can be seen in the Live Statistics panel, the program is continuously computing the velocity of the moving source (v_s) and is comparing that to the velocity of the wave (v_w). Because the source velocity moves slower than the wave propagation velocity ($v_s < v_w$), the wavefronts compress in the direction of travel.

The simulation quantitatively identifies the source as **SUBSONIC**, functioning correctly and displaying a Mach number of $M = 0.42$. With the emitted frequency set to $f = 440$ Hz, the simulation uses its full-precision internal source velocity to calculate:

$$f'_{\text{ahead}} = f \frac{v_w}{v_w - v_s} \approx 755 \text{ Hz}, \quad (3)$$

$$f'_{\text{behind}} = f \frac{v_w}{v_w + v_s} \approx 310 \text{ Hz}. \quad (4)$$

The displayed source velocity of $v_s = 0.83$ is rounded. These results are consistent with the visual output, where compressed waves ahead correspond to a higher perceived frequency, whilst rarefied wavefronts behind correspond to a lower one.

2.3 Sonic Boundary and Wavefront Accumulation ($M = 1$)

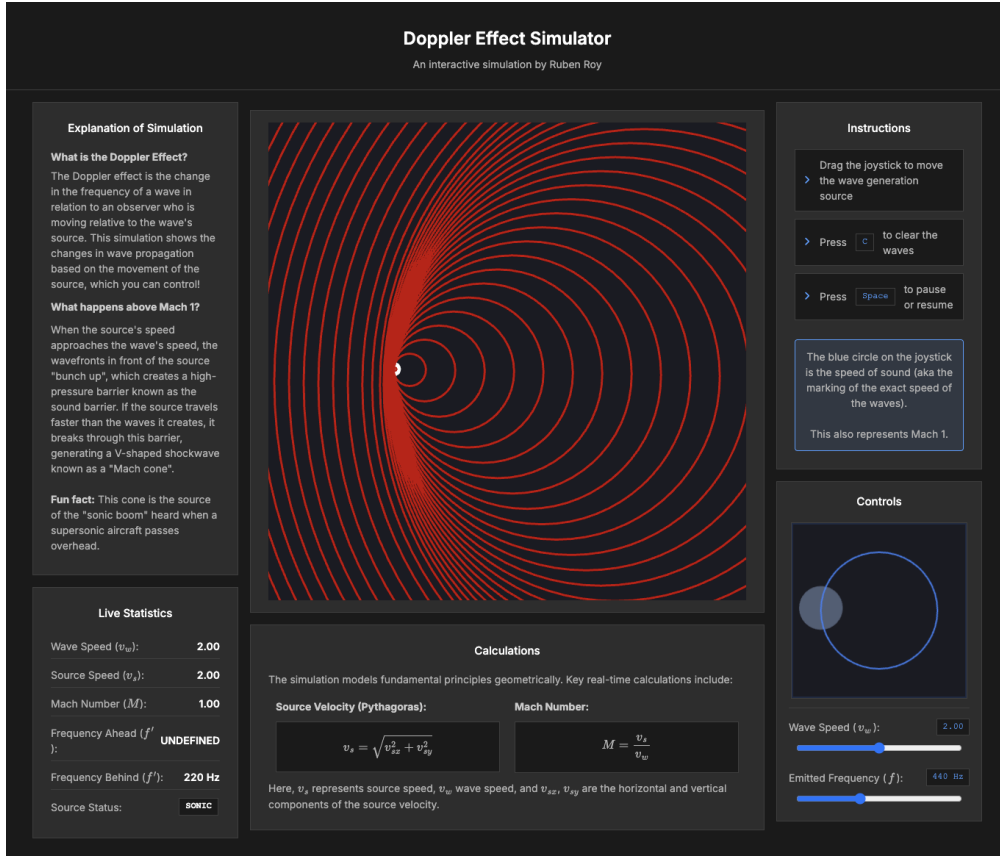


Figure 6. Sonic boundary and wavefront accumulation ($M = 1$).

Note. Screenshot by the author.

When the joystick centre is placed exactly on the blue Mach 1 perimeter, the velocity of the source matches the wave propagation velocity ($v_s = v_w$). This creates maximum wavefront compression at the sound barrier. The simulation's visual output correctly models the overlap and accumulation of waves at the leading edge of the travelling direction of the source. This helps students to visualise the concentration of pressure built by the wavefronts.

At the sonic boundary, $v_s = v_w$, so

$$M = \frac{v_s}{v_w} = \frac{2.00}{2.00} = 1.00. \quad (5)$$

The website correctly identifies the source as **SONIC**, which marks the boundary between subsonic and supersonic motion. With $f = 440$ Hz, the observed frequency behind the source is:

$$f'_{\text{behind}} = f \frac{v_w}{v_w + v_s} = 440 \times \frac{2.00}{2.00 + 2.00} \approx 220 \text{ Hz}. \quad (6)$$

Ahead of the source, the denominator $v_w - v_s$ becomes zero, making the equation undefined at exact equality. This thereby works to show the limit of the classical subsonic Doppler model and the transition to shock-wave behaviour.

2.4 Supersonic Doppler Shift and Mach Cone ($M > 1$)

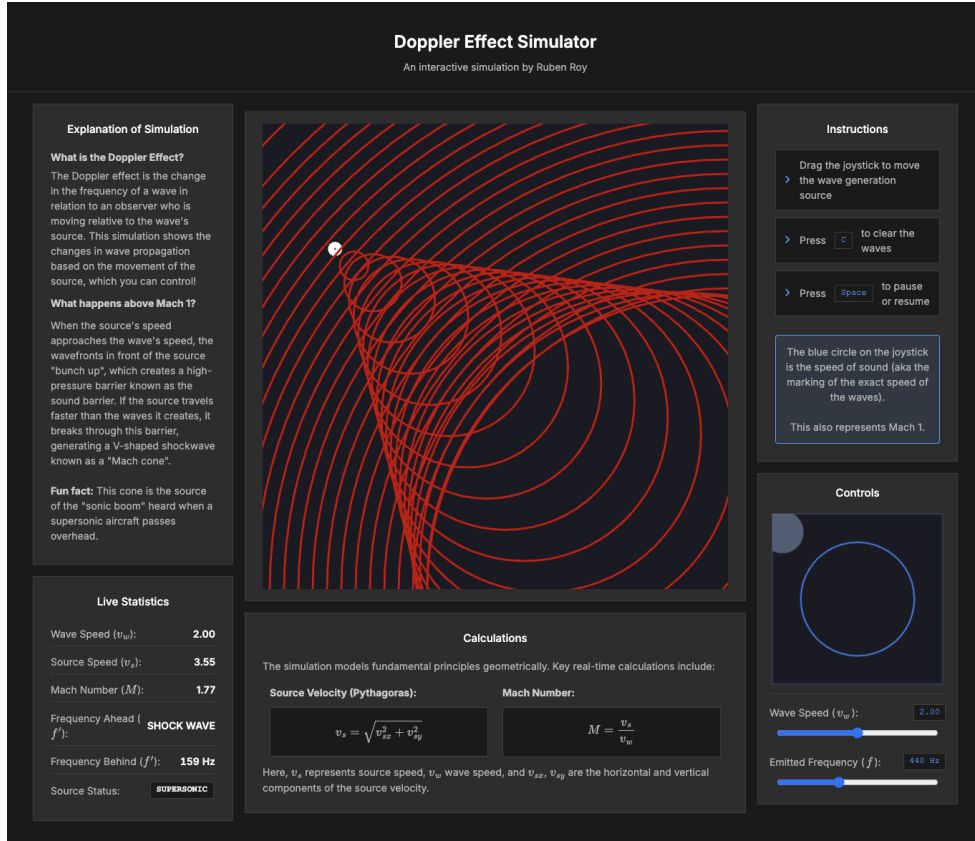


Figure 7. Supersonic Doppler shift and Mach cone ($M > 1$).

Note. Screenshot by the author.

When the source velocity exceeds the wave propagation velocity ($v_s > v_w$), the simulation naturally models Huygens' Principle. The source outruns its secondary wavelets, causing the waves to overlap and coalesce along a V-shaped Mach-cone boundary.

The displayed source velocity is $v_s = 3.55$ and the wave speed is $v_w = 2.00$. Using the full-precision internal source velocity, Mach number is calculated using the relationship

$$M = \frac{v_s}{v_w} = 1.77, \quad (7)$$

correctly identifying the state of the source as **SUPERSONIC**. The Mach angle can be derived from the values:

$$\sin(\mu) = \frac{v_w}{v_s} = \frac{2.00}{3.55}, \quad \mu = \sin^{-1}(0.563) = 34.3^\circ. \quad (8)$$

Because the source is supersonic, the classical frequency-ahead calculation is replaced by **SHOCK WAVE**. Behind the source:

$$f'_{\text{behind}} = f \frac{v_w}{v_w + v_s} = 440 \times \frac{2.00}{2.00 + 3.55} \approx 159 \text{ Hz}. \quad (9)$$

This shows that as the source speed increases, the Mach angle decreases, causing the cone to become narrower and more acute, which is consistent with the increasingly acute V-shape visible in the simulation.

3 Reflection

The digital simulation functions as an effective learning tool for Year 11 Physics students as it makes the otherwise invisible wavefront motion directly visible. Users can vary source speed, wave speed, and emitted frequency continuously and immediately see changes in compression, rarefaction, observed frequency and Mach cone geometry. The red wavefronts contrast with the canvas, which makes the differences in spacing clear. Its quantitative readout calculates the source velocity using Pythagoras and displays the Mach number and observed frequencies, allowing students to compare visual changes with their mathematical cause. The model allows experimentation across subsonic, sonic and supersonic conditions, whilst being consistent with the predictions outlined in the inquiry question.

Although the simulation depicts 2D kinematics effectively, physical inaccuracies regarding acoustics are present. Firstly, the simulation is restricted to a 2-dimensional planar surface, whereas real sound waves propagate spherically in 3D space. Consequently, the model lacks the ability to demonstrate spatial energy dissipation, specifically the Inverse Square Law ($I \propto 1/r^2$) (Serway & Jewett, 2019). Wavefronts are drawn with a uniform thickness, whereas physical wave intensity dissipates as the surface area of the expanding sphere increases.

The accuracy of the simulation is also limited by assuming a homogeneous atmosphere, in which the wave velocity (v_w) remains constant across the canvas. However, atmospheric temperature varies with altitude. Because the speed of sound in air is proportional to the square root of its absolute temperature (Halliday et al., 2014), different portions of the wavefront may travel at different velocities, causing acoustic refraction (Anderson, 2017). For instance, warmer air near the ground increases the speed of the lower wavefront, bending the shock wave upward and distorting the straight, V-shaped geometric lines of the Mach cone depicted by the simulation.

Future versions could improve the physical validity of the simulation by adding an opacity gradient to approximate intensity attenuation under the Inverse Square Law and a vertical temperature gradient to simulate atmospheric refraction. Nevertheless, the model remains effective despite these simplifications thanks to its numerical feedback and interactivity, which helps students identify the transitions between different wave regimes and connect abstract Doppler-effect equations to observable wavefront behaviour.

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